# Effect of floating pile groups on structural response

L'influence de l'ensable de pieux flottés en la reponse structural sismique

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ABSTRACT: The modification of structural period and damping by the dynamic soil-pile group-structure interaction is studied. To model the supporting soil, a uniform stratum over bedrock is considered. The soil-pile interaction is evaluated by impedance functions for isolated piles. The group effect is included by means of interaction factors. An approximate method to evaluate the structural response is applied, which is based on a single replacement oscillator characterized by the effective period and damping of the actual coupled system. Results are evaluated for a typical site of Mexico City.

RÉSUMÉ: On étudie la modification de la période et de l'amortissement structural par l'intéraction du sol-ensamble de piluxstructure. Le modéle étudié se compose d'un depot de sol mui qui repose sur une base rigide. L'interaction sol-pieux est determinée par functions d'impedance pour pieux isolés. L'effet de l'ensamble avec les facteur d'interaction sont includes. Pour évaluer la réponse structurale on utilise une réponse approximative en utilisant un oscoillateur de remplacement avec une base rigide. Les résultants sont appliqués a la Ville de Mexico.

## 1 INTRODUCTION

It is known that the dynamic soil-structure interaction is very important in structures placed on soft soil deposits. To evaluate the soil-foundation response it is necessary to obtain the socalled dynamic stiffness functions.

It has been observed that the also known impedance functions are strongly dependent on the geometrical characteristics of the foundation. In Mexico's valley there are a lot of heavy structures built on deep foundations. Principally, floating piles compose these foundations. Some structures with floating pile foundations have showed large total and differential settlements after strong earthquakes. Some others have collapsed, like those cases occurred on September 1985 earthquake.

In this paper the modification of the relevant natural period and damping ratio of the fixed-base structure due to the dynamic soil–piles–structure interaction is studied. It includes the change on the spectral acceleration. For this propose a soil stratum over bedrock is suggested. The dynamic forces are transmitted to each floating pile through a rigid cap. The structural response is obtained using an equivalent single degree of freedom oscillator.

The soil-pile interaction is evaluated making use of appropriate impedance functions for isolated piles. The group effect is taken into account by use of interaction factors. Horizontal, vertical and rocking oscillations are considered. An approximate method to evaluate the structural response is applied, which is based on the approach of the replacement oscillator commonly used in problems of soil-structure interaction. With this approach, the modified structure in its period and damping is treated like a building with rigid base.

Results are evaluated for a typical site of Mexico City and different structure configurations.

## 2 IMPEDANCE FUNCTIONS

After Lysmer's analogy (1965), it is accepted that the dynamic soil-foundation response is controlled by the impedance functions, representing the springs and dampers by which the soil is replaced.

For each particular time-harmonic excitation of frequency given, the impedance function is defined as the ratio between the force or moment applied on and the resulting displacement or rotation of the foundation.

The impedance functions are complex-valued quantities. Their real and imaginary components are both functions of the exciting frequency. The real component reflects the stiffness and inertia of supporting soil. Its dependence on frequency is attributed solely to the influence that frequency has on inertia, since soil properties are essentially frequency independent. The imaginary component reflects the radiation and material dampings of the system. The former being the result of energy dissipation by waves propagating away from the foundation is frequency dependent, whereas the latter arises mainly from the hysteretic cyclic behavior of soil and is practically frequency independent.

It is common to express the impedance functions by the static stiffness and the impedance coefficients dependent on frequency in the form

$$\widetilde{K}(\omega) = K(k + i\omega c) \tag{1}$$

where K = static stiffness; k = coefficient of stiffness; c = coefficient of damping and  $\omega =$  excitation frequency.

If K and C represent the equivalent soil spring and dashpot, respectively (Figure 1), the impedance function can be written alternatively as the complex expression

$$\widetilde{K}(\omega) = K_m(\omega) + i\omega C_m(\omega) \tag{2}$$

where the subscript m is associated with horizontal, vertical and rocking vibration modes. These parameters are evaluated using the material and the geometrical characteristics of the soil-foundation system.

#### 2.1 Impedance functions of floating piles

The equivalent spring and damping representative of soilfoundation system for isolated piles are determined by

$$K_m = K_m^0 k_m \qquad \& \qquad C_m = \frac{2K_m^0 c_m}{\omega} \tag{3}$$

The dynamic response of piles subjected to lateral and vertical forces as well as moments acting on their head is independent of the pile length. Only the uppermost part of the pile experiences appreciable displacements. It is along this active length that the imposed load is transmitted to the supporting soil.



Figure 1. Equivalent springs and dashpots for the foundation soil.

The equations to evaluate the impedance functions presented in this paper are valid only for "flexible" piles. It is when the pile length is larger than the active length. The pile active length is consider as  $L_c=2d(E_p/E_s)^{0.25}$ . (Gazetas, 1983), where *d* is the pile diameter and  $E_p$  and  $E_s$  are the Young's modulus for pile and soil, respectively. Note that a good majority of real life piles, even some with large diameters in soft soils, would fall into this category. Among exceptions are short piles and caissons.

The static stiffness as well as the stiffness and damping coefficients are presented for an isolated pile embedded into a viscoelastic layer over bedrock. The equations for the impedance functions parameters were originally taken form Gazetas (1983) and presented here after some manipulations. The modes adopted for this analysis are the horizontal (m=h), vertical (m=v) and rocking (m=hr) vibrations. For static stiffness see Equations 4, 5 and 6; for stiffness coefficient Equations 7 and 8, and for damping coefficients Equations 9, 10 and 11.

$$K_h^o = dE_s \left(\frac{E_p}{E_s}\right)^{0.21} \tag{4}$$

$$K_{\nu}^{0} = 1.9 dE_{s} \left(\frac{L_{p}}{d}\right)^{0.67}$$
(5)

$$K_r^o = 0.15d^3 E_s \left(\frac{E_p}{E_s}\right)^{0.75}$$
(6)

$$k_h = k_r = 1 \tag{7}$$

$$k_{\nu} = \begin{cases} 1, \text{ for } L_p / d < 15\\ 1 + \sqrt{\eta}, \text{ for } L_p / d \ge 50 \end{cases}$$

$$\tag{8}$$

$$c_{h} = \begin{cases} 0.8\zeta, \text{ for } \eta \le \eta_{s} \\ 0.8\zeta + 0.175 (E_{p}/E_{s})^{0.17} \eta, \text{ for } \eta > \eta_{s} \end{cases}$$
(9)

$$c_{\nu} = \begin{cases} 0, & \text{for } \eta \le \frac{3.4}{\pi(1-\nu)} \eta_s \\ \frac{0.413}{(1+\nu)} \left(\frac{L_p}{d}\right)^{0.33} \psi \eta^{0.8}, \text{for } \eta > \frac{5.1}{\pi(1-\nu)} \eta_s \end{cases}$$
(10)  
where  $\psi = \left(1 - e^{-(E_p/E_s)(L_p/d)^{-2}}\right)$ 

$$c_r = \begin{cases} 0.25\zeta, \text{ for } \eta \le \eta_s \\ 0.25\zeta + 0.056 (E_p / E_s)^{0.2} \eta, \text{ for } \eta > \eta_s \end{cases}$$
(11)

In the above equations  $\zeta =$  material soil damping; v = Poisson's modulus;  $\eta = \pi d/\beta_s$  is the normalized frequency and  $\eta_s = \pi d/2H_s$  and  $\eta_p = \pi d\alpha_s/2H_s\beta_s$  represent the dominant dimensionless frequencies of the layer by transversal and vertical vibrations waves, respectively.  $\alpha_s$  and  $\beta_s$  are the P- and S-wave velocities, respectively. For vertical vibrations in the range 15 <  $L_p/d < 50$  and  $\eta_p < \eta < 1.5\eta_p$ , a linear interpolation is admitted. The geometrical parameters included in Equations 4 to 11 are shown in Figure 2.

The formulas showed before are reasonably accurate, as they are basically curve fits to rigorous numerical results.

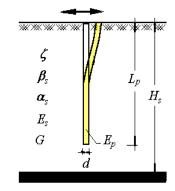


Figure 2. Floating pile under horizontal vibrations.

## 3 IMPEDANCE FUNCTIONS OF FLOATING PILES FOR MEXICO CITY

Analyzing the typical properties of the subsoil of Mexico City and the common geometrical and material characteristics of floating pile foundations built there, and based on Equations 4 to 11, we have constructed plots to determine the necessary parameters to calculate the impedance functions. The results of this analysis are shown in Figures 3, 4 and 5 for static stiffness, damping coefficients and stiffness coefficients, respectively. In each figure, results for the horizontal, vertical and rocking modes are given.

The Young's modulus for soil was taken between 10 and 10,000 kg/cm<sup>2</sup>; the Poisson's modulus and the material damping were considered as  $\frac{1}{2}$  and 5%, respectively. The diameter of piles was adopted between 20 and 50 cm and length no more than 70 m. The good majority of floating piles in Mexico City are constructed in concrete, so it was assumed a Young's modulus for pile as 230,000 kg/cm<sup>2</sup>, approximately.

In Figure 3b,  $L_{p}/d = 10$ , 50, 100, 250 and 500. Note that in Figure 4 the vertical damping coefficient depends on the slenderness ratio, besides Young's modulus of soil and dimensionless frequency.

The variation of the stiffness coefficient for vertical vibration is shown in Figure 5. For the others vibrations modes, this coefficient is considered unity.

## 4 INTERACTION FACTORS FOR PILE GROUPS

A simple analytical solution is adopted for computing the dynamic impedance of floating rigidly capped pile groups with due consideration to pile-soil-pile interaction. The method, proposed by Dobry & Gazetas (1988), introduces some sound physical approximations and considers the interference of cylindrical wave fields originating along each pile shaft and spreading radially outward. The predictions of the simple method compare well with rigorous numerical solutions.

Poulos (1968, 1971) superposition procedure originally developed for statically loaded pile groups, is also valid for the dynamic problem. Therefore, considering two identical piles, separated by a distance *S* between axes, the effect of the vibration pile on the response of the other pile, can be conveniently expressed through the dynamic interaction factor  $\alpha$ , which is a function of frequency. For vertical and laterally oscillating piles, the approximate expressions for interaction function are,

$$\alpha_{v} \approx \left(\frac{S}{r_{0}}\right) e^{-\zeta \omega S/\beta_{s}} e^{-i\omega S/\beta_{s}}$$
(12)

$$\alpha_h(\theta^\circ) \approx \alpha_h(0^\circ) \cos^2 \theta + \alpha_h(90^\circ) \sin^2 \theta \tag{13}$$

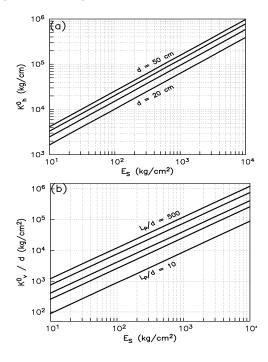
in which,

$$\alpha_h(0^\circ) \approx \left(\frac{S}{r_0}\right)^{-0.5} e^{-\zeta \omega S/\beta_L} e^{-i\omega S/\beta_L} \qquad \& \qquad \alpha_h(90^\circ) \approx \alpha_v$$
(14)

where  $\alpha_{\nu}$  and  $\alpha_h$  are the interaction factors for vertical and horizontal vibrations, respectively, and  $\beta_L = 3.4\beta_s/\pi/(1-\nu)$ .

It is assumed that no interaction takes place due to the rotational deformation of each pile under rocking vibrations. Such deformation is felt only a few diameters down from the pile head, and produces a rapidly decaying stress around the pile.

For each vibration mode, with Equation 12 and 13 a matrix interaction factor is constructed for the pile group. The force on each pile is obtained solving a complex system of algebraic equations. It is composed by the matrix interaction factors, the impedance function for an isolated pile and a vector of unity displacements (Aguilar & Avilés, 1999). The group pile impedance function is evaluated by the ratio between the forces on piles and the displacement associated.



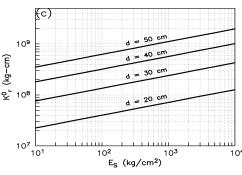


Figure 3. Static stiffness for floating pile on Mexico City under horizontal (a), vertical (b) and rocking (c) vibrations.

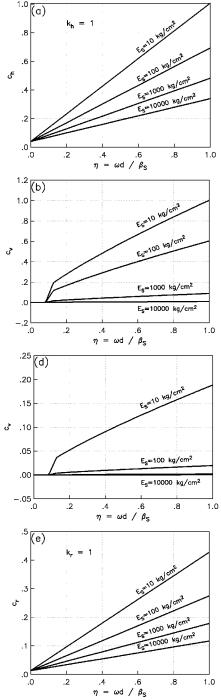


Figure 4. Damping coefficients for floating pile on Mexico City under horizontal (a), vertical with  $L_{p'}/d = 50$  (b), vertical with  $L_{p'}/d = 500$  (d) and rocking (c) vibrations.

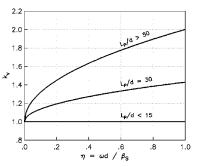


Figure 5. Stiffness coefficients for floating pile on Mexico City under vertical vibrations.

#### 5 STRUCTURAL RESPONSE

When the soil flexibility is included in the dynamic structural response, the inertial interaction effects should be evaluated. The response system shows an enlargement of its natural structural period and a change in its structural damping. The modified parameters can be considered as effective period and damping (Avilés et al. 1992). The proposed method is based on the determination of these equivalent parameters.

Under harmonic excitation, with time dependence given by  $e^{iwt}$ , the dynamic equilibrium equation can be expressed as

$$\left[\mathbf{K}_{s}+i\omega\mathbf{C}_{s}-\omega^{2}\mathbf{M}_{s}\right]X_{s}=-\ddot{X}_{0}\mathbf{M}_{0}$$
(16)

where  $K_s$ ,  $C_s$  and  $M_s$  are the stiffness, damping and mass matrices of the system, respectively.  $M_0$  represents a load vector.

To obtain the period and damping with the dynamic interaction effect, it is necessary to relate the real and imaginary parts of the solution in Equation 16 with the real and imaginary parts of the equation of an equivalent single oscillator with rigid base. It is necessary to consider both systems in resonance.

By this procedure, the effective structural period can be represented by

$$\tilde{T}_{e} = (T_{e}^{2} + T_{h}^{2} + T_{r}^{2})^{1/2}$$
(17)

in which

$$T_h = 2\pi \left(\frac{M_e}{K_h}\right)^{1/2} \quad \& \quad T_r = 2\pi \left(\frac{M_e(H_e + D)}{K_r}\right)^{1/2}$$
(18)

where  $T_e$  is the structural fundamental period with rigid base;  $T_h$  and  $T_r$  are the natural periods on horizontal and rocking vibrations, respectively; D is the representative depth foundation and  $M_e$  and  $H_e$  are the structure equivalent mass and height, respectively. Note that  $K_h$  and  $K_r$  are the equivalent springs of soil taken from the impedance functions.

The determination of the effective period follows a iterative process, since the natural periods in horizontal and rocking vibrations are unknown.

On the other hand, by equating the imaginary parts of the equations mentioned before neglecting damping terms of second order, an approximation of the effective damping empirically calibrated with the rigorous solution is given by

$$\widetilde{\xi}_{e} = \xi_{e} \left(\frac{T_{e}}{\widetilde{T}_{e}}\right)^{3} + \frac{\zeta_{h}}{1 + 2\zeta_{h}^{2}} \left(\frac{T_{h}}{\widetilde{T}_{e}}\right)^{2} + \frac{\zeta_{r}}{1 + 2\zeta_{r}^{2}} \left(\frac{T_{r}}{\widetilde{T}_{e}}\right)^{2}$$
(19)

where  $\zeta_h$  and  $\zeta_r$  represent the damping on horizontal and rocking modes, respectively, evaluated as

$$\zeta_h = \frac{\omega C_h}{2K_h} \qquad \& \qquad \zeta_r = \frac{\omega C_r}{2K_r} \tag{20}$$

Note that  $C_h$  and  $C_r$  represent the equivalent soil dashpot and are also taken from the impedance functions.

#### 6 RESULTS

With the procedure presented above a 56 m soil deposit is studied. Its properties are  $\beta_s = 67.7$  m/s, v = 0.45 and  $\zeta = 5$ %. The analyzed structures show between 0.2 and 5 s of fundamental structural periods and 5% of material damping. The structure foundation is partially compensated with 64 floating piles. The plan dimension of foundation is 20 x 20 m and its piles have 40 cm diameter. This configuration is shown on Figure 6.

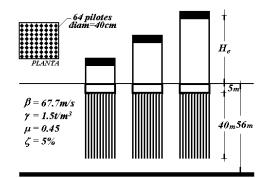
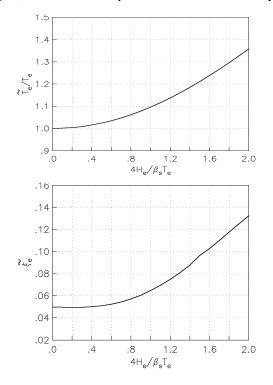


Figure 6. Soil-foundation-structure system

The results are presented in Figures 7 and 8. In figure 7 the enlargement of period with respect the fixed base original structural period is shown. In figure 8 the increase of damping by dynamic interaction effect is presented. For both, 7 and 8 Figures, the abscissa axe represents the interaction intensity.



## 7 CONCLUSIONS

Charts to obtain impedance function for floating piles foundation in Mexico City are presented. A simple analytical solution to consider floating pile groups is included. An approximate technique to compute the response of the soil-foundationstructure is presented. Effective period and damping of structures with floating piles in soft soil deposit on Mexico City are presented.

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